AN ROINN OIDEACHAIS

LEAVING CERTIFICATE EXAMINATION, 1996

APPLIED MATHEMATICS – ORDINARY LEVEL

FRIDAY, 21 JUNE – MORNING, 9.30 to 12.00

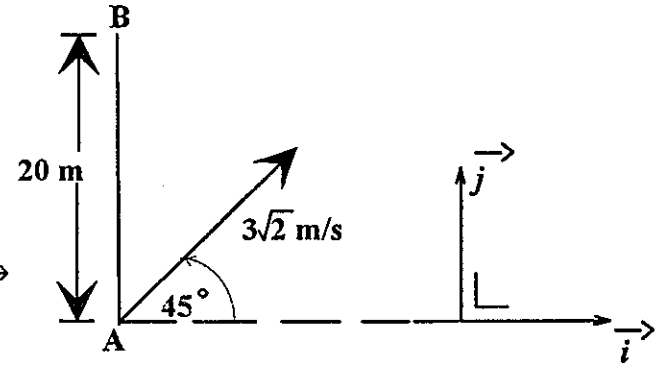
Six questions to be answered. All questions carry equal marks.
Mathematics Tables may be obtained from the Superintendent.
Take the value of g to be 10 m/s^2.
$\vec{i}$ and $\vec{j}$ are unit perpendicular vectors in the horizontal and vertical directions, respectively.

Marks may be lost if necessary work is not clearly shown or you do not indicate where a calculator has been used.

1. A train, starting from rest, is uniformly accelerated to reach a speed of v m/s in 100 s over 500 m.
The speed of v m/s is maintained for a further 1500 m and the train is then brought to rest in the next 250 m under uniform retardation.

(i) Find the value of v.
(ii) Find the acceleration and the retardation of the train.
(iii) Find the total time for the journey.
(iv) Draw an accurate velocity-time graph of the motion of the train.

2. At a particular instant on a football pitch, player A is 20 m due South of player B.
Player A is running with a speed of $3\sqrt{2}$ m/s in a North-easterly direction.
Player B is running with a speed of 5 m/s to intercept A. Let the velocity of B be $x\,\vec{i} + y\,\vec{j}$ m/s.



(i) Write down, in terms of $\vec{i}$ and $\vec{j}$, the velocity of A.

(ii) If B is to intercept A, find the value of x and the value of y.

(iii) Find the time taken for B to catch A, correct to one place of decimals.

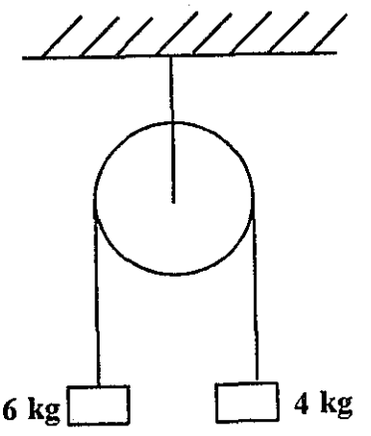
3. Two missiles, A and B, are fired simultaneously out to sea from the top of a cliff 90m above sea level.
Missile A is fired horizontally with a speed of 100 m/s.
Missile B is fired upwards with a speed of 80 m/s at an angle of 45° to the horizontal.
Each missile strikes the sea.

Show that the time of flight of missile A is one-third the time of flight of missile B.

Calculate the distance between the points at which the missiles strike the sea, correct to the nearest metre.

OVER→

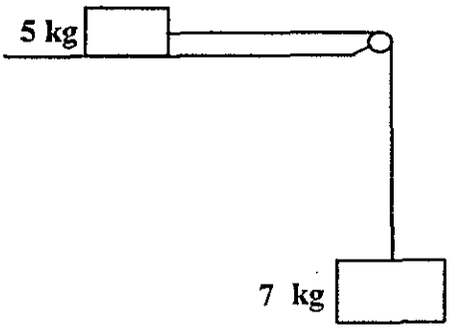
4. *(a)* Two particles of masses 6 kg and 4 kg are connected by a light inextensible string passing over a smooth fixed pulley. The particles are released from rest.



(i) Write down the equation of motion for each particle.

(ii) Find the common acceleration of the particles and the tension in the string.

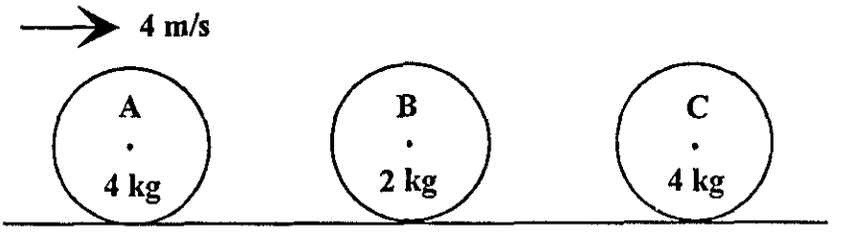
(b) A particle of mass 5 kg resting on a rough horizontal table is connected by a light inextensible string passing over a smooth pulley at the edge of the table to a particle of mass 7 kg hanging freely.



The particles are released from rest. The common acceleration of the particles is 2 m/s^2.

Find the value of μ, the coefficient of friction between the 5 kg mass and the table.

5. Three smooth spheres A, B and C lie at rest in a straight line on a smooth horizontal table.



The mass of sphere A is 4 kg, of sphere B is 2 kg and of sphere C is 4 kg.

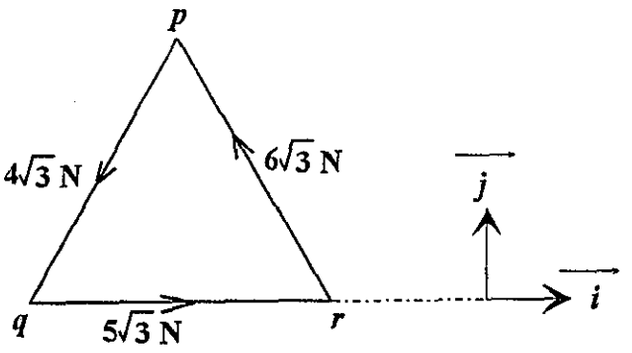
A is projected with a velocity of 4 m/s so as to collide directly with B.
The coefficient of restitution for the collision is ½.
Find the velocity of A and the velocity of B after this collision.

B now moves on to collide directly with C and as a result of this collision B is brought to rest.
Find the coefficient of restitution for this impact.

A then strikes B again. Show that this is the final collision between the spheres.

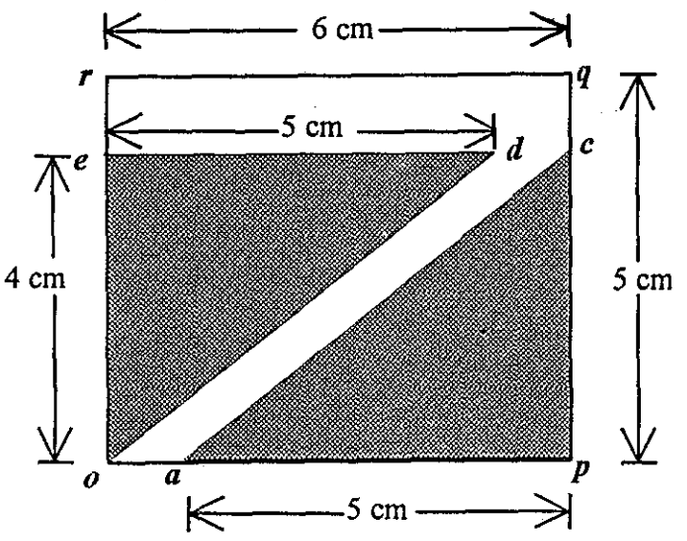
6. *(a)* *pqr* is a triangle with $|pq| = |qr| = |rp|$.
Forces $4\sqrt{3}$ N, $5\sqrt{3}$ N,
and $6\sqrt{3}$ N act along the sides
[*pq*], [*qr*] and [*rp*] as shown.

Express each force in terms of
$\vec{i}$ and $\vec{j}$ and find their resultant.



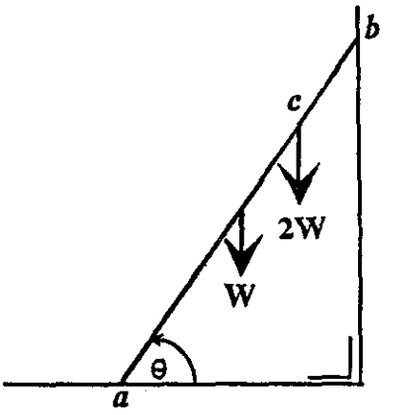
(b) *opqr* is a uniform
rectangular lamina
of length 6 cm and width
5 cm. The figure 7
is made by cutting out
from the lamina the two
congruent right angled
triangles *oed* and *apc*.

Find the distance of the
centre of gravity of the
figure 7 from the point *o*.
Give your answer correct
to two decimal places.



7. *(a)* A uniform ladder, [*ab*], of weight W,
is placed with end *b* against a rough
vertical wall and end *a* on rough horizontal
ground. The ladder is of length 4 m. The
coefficient of friction μ is the same at *a* and *b*.

A person of weight 2W begins to climb the ladder.
When the person reaches the point *c*, the ladder is
on the point of slipping with $\tan\theta = \frac{3}{2}$ and $|ac| = 3$ m.

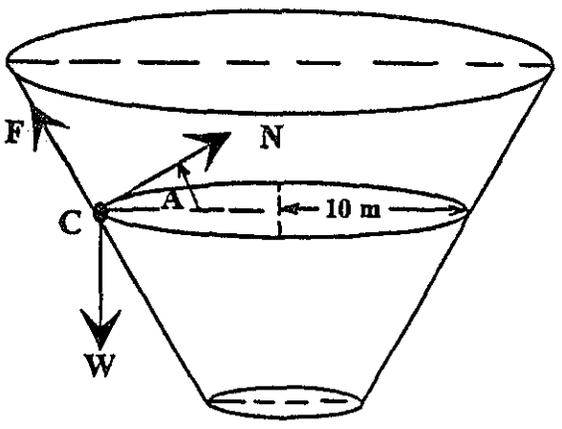


(i) Draw a diagram showing all the forces
acting on the ladder.

(ii) Find the value of μ, correct to one place of decimals.

OVER→

8. A motorcyclist on a motorcycle describes a horizontal circle of radius 10 m on a "wall of death". The weight of the motorcyclist together with the weight of the motorcycle is denoted by W. The three forces acting at C are shown where C denotes the motorcyclist and motorcycle. The angle between the normal reaction N and the horizontal is A, where $\sin A = \frac{3}{5}$. The force due to friction is denoted by F.



The constant speed of the motorcyclist is v m/s.

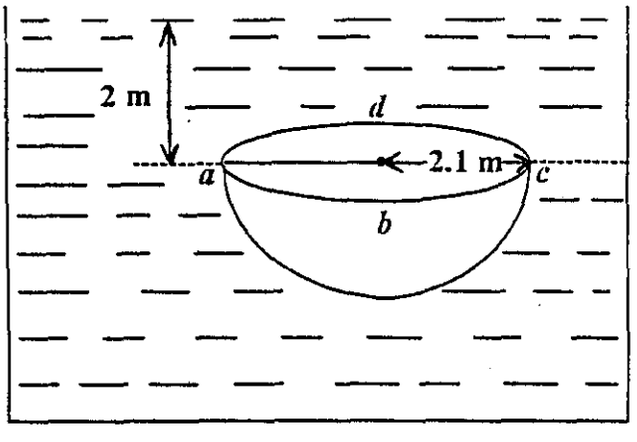
The coefficient of friction between the tyres of the motorcycle and the wall is $\frac{1}{2}$.

If C is about to slip down the wall, show that

(i) $N = W$

(ii) $v = 5\sqrt{2}$ m/s.

9. *(a)* A solid hemisphere of radius 2.1 m is in equilibrium in a tank of water. The plane face *abcd* is in a horizontal position at a depth of 2 m below the surface of the water.



Find

(i) the thrust downwards on the face *abcd*

(ii) the buoyancy force

(iii) the thrust upwards on the curved surface.

Take $\pi = \frac{22}{7}$.

Density of water = 1000 kg/m^3.